Stationary and uniform entanglement distribution in qubit networks with quasi-local dissipation

Morteza Rafiee[1,2*], Cosmo Lupo[2†], Hossein Mokhtari[1‡] and Stefano Mancini[2§]
[1] *Department of Physics, Yazd University, Pajoohesh St, Safaieh, 89195-741 Yazd, Iran and*
[2] *School of Science and Technology, University of Camerino, I-62032 Camerino, Italy*

We consider qubit networks where adjacent qubits besides interacting via XY-coupling, also dissipate into the same environment. The steady states are computed exactly for all network sizes and topologies, showing that they are always symmetric under permutation of network sites, leading to a uniform distribution of the stationary entanglement across the network. The maximum entanglement between two arbitrary qubits is shown to depend only on the total number of qubits in the network, and scales linearly with it. A possible physical realization by means of an array of doped cavities is discussed for the case of a linear chain.

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I. INTRODUCTION

It has been understood for a long time that entanglement represents a quintessential and characteristic trait of quantum mechanics [1]. Quantum phenomena are by now very well known to be key resources for communication and computation [2], and it has been recently questioned whether they play a functional role in certain biological processes [3, 4]. Due to its fragility under environment induced decoherence, entanglement is commonly considered to be an elusive physical phenomenon that can observed only in the most elementary systems and on the shortest time scales. Nevertheless, together with a variety of entanglement preserving mechanisms that have been put forward [5–12], the idea is now spreading that it can persist on relatively long time scales, even in a noisy environment, if suitable conditions are fulfilled. To achieve stationary entanglement in spin systems it is sufficient to have quasi-local (two-body interaction) Hamiltonian and local dissipation [7], or local Hamiltonian and quasi-local (two-body) dissipation [8–10]. Till now, these two possibilities have been studied separately or for systems composed of a small number of qubits. The main aim of this paper is to study the effects of both quasi-local interaction and dissipation in a system composed of an arbitrary number of qubits. Our goal is to determine general conditions for stationary entanglement and characterize its distribution among qubits.

We consider a family of models of quantum networks consisting of n qubits with onsite energy and XY interaction between adjacent qubits. Moreover, a non-Hamiltonian dynamical term is added within the quantum master equations formalism [13]. The latter describes quasi-local dissipation coupling of adjacent qubits, which can be understood as arising from the coherent damping to the same, zero-temperature, bosonic environment. We compute the steady states for any size and network topology. This allows us to characterize the given model of quasi-local dissipation as a means for distributing stationary entanglement over a generic network. We found that the steady states are largely independent on the dynamical features of the model, like the strength of the onsite energy or of the XY interaction, and on the network topology. The steady states are always symmetric under permutation of the network sites, yielding a uniform distribution of entanglement across the network. In particular the maximum attainable entanglement between any pair of qubits, measured by concurrence [14], equals $2/n$ ebits and is independent on the relative position of the two qubits. Furthermore, we investigate the steady-state entanglement as a function of the initial state. Finally, for the special case in which the network reduces to a chain we discuss a possible physical realization by means of an array of doped optical cavities.

The paper proceeds as follows. In Sec. II we introduce the model of the qubit network; in Sec. III we compute the steady states of the qubit network; the distribution of stationary entanglement across the network is discussed in Sec. IV; in Sec. V a possible physical realization is introduced; Sec. VI is devoted to conclusions.

II. THE NETWORK MODEL

We consider a network of n qubits defined by a connected graph G, with vertices $V(G)$ and edges $E(G)$, where a qubit system is sitting at each vertex of the graph and the edges identify two body interactions between the qubits. Let

$$A[G]_{k,l} = \begin{cases} 0 & \text{if} \quad k,l \notin E(G) \\ 1 & \text{if} \quad k,l \in E(G) \end{cases} , \tag{1}$$

be the (symmetric) adjacency matrix of such a graph.

Then, by considering on site energy and XY-

*m.rafiee178@gmail.com
†cosmo.lupo@unicam.it
‡phmh.mokhtari@yazduni.ac.ir
§stefano.mancini@unicam.it

interaction, the network Hamiltonian is defined as

$$H = \sum_{k=1}^{n} \omega_k \sigma_k^\dagger \sigma_k + \frac{1}{2} \sum_{k \neq l = 1}^{n} [A(G)]_{k,l} J_{k,l} \left(\sigma_k^\dagger \sigma_l + \sigma_l^\dagger \sigma_k \right) , \tag{2}$$

where ω_k and $\sigma_k^\dagger$, σ_k denote respectively the energy and the raising, lowering operators of the kth qubit. Furthermore $J_{k,l}$ is the coupling strength between qubits k and l.

We assume that the dynamics of the network is described by the master equation ($\hbar = 1$)

$$\dot{\rho} = \mathcal{L}(\rho) , \tag{3}$$

where

$$\mathcal{L}(\rho) = -i[H, \rho] + \mathcal{D}(\rho) . \tag{4}$$

Here, adjacent qubits interact both directly through H and indirectly through a non-Hamiltonian term given in the Gorini-Kossakowski-Sudarshan-Lindblad form [13],

$$\mathcal{D}(\rho) = \sum_{k \neq l = 1}^{n} [A(G)]_{k,l} \mathcal{D}_{k,l}(\rho) , \tag{5}$$

where

$$\mathcal{D}_{k,l}(\rho) = \frac{\gamma_{k,l}}{2} \left(2 L_{k,l} \rho L_{k,l}^\dagger - L_{k,l}^\dagger L_{k,l} \rho - \rho L_{k,l}^\dagger L_{k,l} \right) , \tag{6}$$

with $\gamma_{k,l} > 0$ and

$$L_{k,l} = \sigma_k + \sigma_l . \tag{7}$$

The non-Hamiltonian term $\mathcal{D}(\rho)$ describes a Markovian damping process in which pairs of adjacent qubits coherently decay into the same zero-temperature bosonic bath, with decay rates $\gamma_{k,l}$.

III. THE NETWORK STEADY STATES

A pure state $|\psi\rangle \in \mathbb{C}^{2 \otimes n}$ is a steady state of the network if it satisfies $\mathcal{L}(|\psi\rangle\langle\psi|) = 0$. A characterization of the pure steady states of an open quantum system undergoing a Markovian dynamics [13] has been provided in [11]. Following [11], a pure steady state of our network model is characterized by the conditions:

1. $[A(G)]_{k,l} L_{k,l} |\psi\rangle = \lambda_{k,l} |\psi\rangle$, for all k, l, with $\lambda_{k,l} \in \mathbb{C}$;

2. $\left[iH + \frac{1}{2} \sum_{k \neq l} [A(G)]_{k,l} \gamma_{k,l} L_{k,l}^\dagger L_{k,l} \right] |\psi\rangle = \lambda |\psi\rangle$, with $\lambda \in \mathbb{C}$;

3. $\mathrm{Re}(\lambda) = \frac{1}{2} \sum_{k \neq l} [A(G)]_{k,l} \gamma_{k,l} |\lambda_{k,l}|^2$, where $\mathrm{Re}(\lambda)$ denotes the real part of λ.

To compute the pure steady states of the network we first notice that the operators $L_{k,l} = \sigma_k + \sigma_l$ are nilpotent, hence admitting only vanishing eigenvalues. Thus, condition 1 reads

$$[A(G)]_{k,l} (\sigma_k + \sigma_l) |\psi\rangle = 0 . \tag{8}$$

A pure state can be expanded in the standard basis,

$$|\psi\rangle = \sum_{a_1, \ldots, a_n = 0,1} \psi_{a_1, \ldots, a_n} |a_1, \ldots, a_n\rangle , \tag{9}$$

where $\sigma_k^\dagger \sigma_k |a_1, \ldots, a_n\rangle = a_k |a_1, \ldots, a_n\rangle$. For any pair of adjacent sites, k, l, Eq. (8) implies

$$\begin{aligned} 0 = & \sum_{a_k, a_l = 0,1} \psi_{a_1, \ldots 1_k, \ldots, a_l, \ldots, a_n} |a_1, \ldots, 0_k, \ldots, a_l, \ldots, a_n\rangle \\ & + \psi_{a_1, \ldots, a_k, \ldots, 1_l, \ldots, a_n} |a_1, \ldots, a_k, \ldots, 0_l, \ldots, a_n\rangle , \end{aligned} \tag{10}$$

where the notations 0_k, 1_k, 0_l, 1_l are used to indicate that $a_k = 0$, $a_k = 1$, $a_l = 0$, $a_l = 1$, respectively. This in turn yields

$$\psi_{a_1, \ldots, 1_k, \ldots, 0_l, \ldots, a_n} + \psi_{a_1, \ldots, 0_k, \ldots, 1_l, \ldots, a_n} = 0 , \tag{11}$$

$$\psi_{a_1, \ldots 1_k, \ldots, 1_l, \ldots, a_n} = 0 . \tag{12}$$

If there are no isolated points in the network, these conditions imply that the pure steady states can contain at most one excitation. They can be written as superposition of the network vacuum, $|0\rangle \equiv |0_1, \ldots, 0_n\rangle$, and the single excitation states, $|k\rangle \equiv |0_1, \ldots, 1_k, \ldots, 0_n\rangle$ for $k = 1, \ldots, n$. To simplify the notation we can expand the pure steady states as

$$|\psi\rangle = \alpha |0\rangle + \beta \sum_{k=1}^{n} \psi_k |k\rangle , \tag{13}$$

where the condition (11) implies

$$[A(G)]_{k,l} (\psi_k + \psi_l) = 0 . \tag{14}$$

We can now distinguish two situations according to the network topology:

(i) If the network does not contain cycles or it contains only cycles with a even number of edges, the solution is given by $\psi_k = (-1)^{n_k} \psi_1$, where n_k is the number of edges connecting the kth site with the first one;

(ii) Otherwise, if the network contains cycles with odd number of edges, the only solution is obtained by putting $\psi_k = 0$ for all k.

In conclusion, we get that the pure steady states have the form

$$|\psi\rangle = \alpha |0\rangle + \beta |\aleph\rangle , \tag{15}$$

where

$$|\aleph\rangle = \frac{1}{\sqrt{n}} \sum_{k=1}^{n} (-1)^{n_k} |k\rangle \,. \tag{16}$$

The coefficients $\alpha, \beta \in \mathbb{C}$ are arbitrary if the network topology fulfills (i). On the other hand, we have to put $\beta = 0$ if (ii) holds. Being interested in the distribution of stationary entanglement, in the following we assume that (i) is verified.

Then, the condition 2 reads

$$iH\left(\alpha|0\rangle + \beta|\aleph\rangle\right) = \lambda\left(\alpha|0\rangle + \beta|\aleph\rangle\right) , \tag{17}$$

which may have two independent solutions:

- The first solution is obtained for $\alpha = 0$ under the conditions

$$i\lambda = \omega_k - \sum_l [A(G)]_{k,l} J_{k,l} \,, \tag{18}$$

  for all $k = 1, \ldots, n$.

- The second solution is obtained for $\beta = 0$, with $\lambda = 0$;

In the degenerate case, $\omega_k - \sum_l [A(G)]_{k,l} J_{k,l} = 0$, there exists a two-dimensional steady subspace $\mathcal{H}_s = \text{span}\{|0\rangle, |\aleph\rangle\}$. Otherwise if $\omega_k - \sum_l [A(G)]_{k,l} J_{k,l} = const. \neq 0$, the only pure steady states are $|0\rangle$ and $|\aleph\rangle$. Finally, we notice that the condition 3 is satisfied in both cases, since $\text{Re}(\lambda) = 0$.

Furthermore, it is worth noticing that it could be possible that other mixed steady states exist, which are not in the convex hull of pure steady states.

The steady states of our models do fulfill $[A(G)]_{k,l} L_{k,l}|\psi\rangle = 0$. The steady states satisfying such a property are called *dark states*. A uniqueness theorem for the dark states has been provided in [11], for a system admitting a subspace of dark states. We can hence apply this theorem in the degenerate case, in which the subspace $\mathcal{H}_s = \text{span}\{|0\rangle, |\aleph\rangle\}$ is a subspace of dark states. According to this result, if there exists no subspace S with $S \perp \mathcal{H}_s$ such that $[A(G)]_{k,l} L_{k,l} S \subseteq S$, then the only mixed steady-states are in the convex hull of $\mathcal{H}_s$. It is easy to show that a subspace with such a property does not exist for our models. To show that it is sufficient to notice that since the operators $L_{k,l}$ are nilpotent, a subspace S which is invariant under the action of all the $L_{k,l}$'s must necessarily include $|0\rangle$ or $|\aleph\rangle$. Thus, S cannot be orthogonal to $\mathcal{H}_s$.

IV. STEADY-STATE ENTANGLEMENT DISTRIBUTION

In this section we study the steady-state entanglement between two arbitrary network qubits. Due to the symmetric form of (16), the extension of the analysis of multiqubit entanglement is straightforward.

Since we are interested in the distribution of steady-state entanglement across the network, in the following we will restrict to the case in which both condition (ii) and Eq. (18) are fulfilled. Under this hypothesis, the reduced steady state of two arbitrary qubits at site k and j of the network is necessarily of the form

$$\rho_s^{k,j} = \left(1 - \frac{2p}{n}\right) |00\rangle\langle 00| + \frac{2p}{n} |\Psi^{k,j}\rangle\langle\Psi^{k,j}| \,, \tag{19}$$

where $|\Psi^{k,j}\rangle = \left[|10\rangle + (-1)^{n_j - n_k}|01\rangle\right]/\sqrt{2}$ is a maximally entangled state. It is worth noticing that the reduced steady-state contains only one free parameter, p. Such a parameter is determined by the initial state of the network. To fix the ideas, we consider the concurrence [14] as an entanglement measure for the reduced state of the two qubits. The stationary concurrence of the two-qubit reduced state is as well a function of p and the total number of qubits, namely

$$C_s = \frac{2p}{n} \,. \tag{20}$$

In order to evaluate p for a given initial state of the network, let us notice that

$$\langle\aleph|\dot{\rho}(t)|\aleph\rangle = \frac{1}{2} \sum_{k \neq l = 1}^{n} [A(G)]_{k,l} \gamma_{k,l} \langle\aleph| L_{k,l} \rho(t) L_{k,l}^{\dagger} |\aleph\rangle \,, \tag{21}$$

where we have used the fact that $[A(G)]_{k,l} L_{k,l}|\aleph\rangle = 0$, and that the vectors $L_{k,l}^{\dagger}|\aleph\rangle$ are superpositions of states containing two or more excitations. Moreover, we remark that the total number of excitations in the network cannot increase under the evolution dictated by the master equation (4). Therefore, we conclude that if the initial state $\rho(0)$ contains up to one excitation, it follows that $\langle\aleph| L_{k,l} \rho(t) L_{k,l}^{\dagger} |\aleph\rangle = 0$ for any $t \geq 0$, which in turn yields that the quantity $\langle\aleph|\rho(t)|\aleph\rangle$ is a constant of motion. Eventually we get $p = \langle\aleph|\rho(0)|\aleph\rangle$, which allows us to compute the steady-state parameter p for any initial state of the network, provided it contains up to one excitation. Let us further explore this setting by assuming that the network is initialized in a state containing a single excitation over m qubits, that is, $|\psi(0)\rangle = \sum_{j=1}^{m} \alpha_j |k_j\rangle$. The maximum stationary concurrence of the two-qubit reduced state is hence obtained by maximizing $p = |\langle\aleph|\psi_0\rangle|^2$. It follows that the optimal choice for the initial state is

$$|\psi(0)\rangle = \frac{1}{\sqrt{m}} \sum_{j=1}^{m} (-1)^{n_{k_j}} |k_j\rangle \,, \tag{22}$$

yielding $p = m/n$.

We define $C_s(1, m)$ as the maximum stationary concurrence that can be achieved by preparing the network into an initial state containing up to 1 excitation over m qubits. We then have obtained that

$$C_s(1, m) = \frac{2m}{n^2} \,. \tag{23}$$

To go beyond the single-excitation setting, we have analyzed numerically the achievable stationary concurrence for initial states containing more than one excitation. By defining $C_s(N,m)$ as the maximum stationary concurrence for an initial state containing up to N excitations over m qubits, our numerical investigations suggest to conjecture that

$$C_s(N,m) \leq C_s(1,m)\,, \tag{24}$$

where the optimal network initial state is as in Eq. (22), that is, a single-excitation initial state is sufficient to achieve the overall maximum concurrence for a given m.

V. PHYSICAL REALIZATION

We sketch here a possible physical realization by an array of n doped cavities coupled via optical fibers (see, e.g., [15, 16]). Actually, we restrict our attention to the case in which the network is a linear chain with open boundary conditions. The case of periodic boundary conditions can be analyzed in a similar way. In the case of a linear chain, the model generalizes that introduced in [10] where the Hamiltonian term is dropped.

Each cavity is doped with a two-level atom and is coupled by optical fibers to the next-nearest cavities. We denote as c_k, $c_k^\dagger$ the ladder operators of the kth cavity, coupled to the levels $|g\rangle_k$, $|e\rangle_k$ of the kth atom. Neighboring cavities are in turn coupled through a single fiber mode, having ladder operators a_k, $a_k^\dagger$. Furthermore, we assume that the kth fiber mode interacts with its bosonic environment, described by a collection of operators $\{b_{k,j}, b_{k,j}^\dagger\}$. The Hamiltonian of the system in the rotating wave approximation is given by

$$H = H^{free} + H^{int}\,, \tag{25}$$

where

$$\begin{aligned} H^{free} =& \sum_{k=1}^{n} \omega_k^c c_k^\dagger c_k + \sum_{k=1}^{n} \omega_k^a |e\rangle_k\langle e| + \sum_{k=1}^{n-1} \omega_k^f a_k^\dagger a_k \\ &+ \sum_{k=1}^{n-1}\sum_{j} \omega_{k,j}^e b_{k,j}^\dagger b_{k,j}\,, \end{aligned} \tag{26}$$

and

$$\begin{aligned} H^{int} =& \sum_{k=1}^{n} f_k \left(c_k^\dagger |g\rangle_k\langle e| + H.c.\right) \\ &+ \sum_{k=1}^{n-1} J_k \left[a_k^\dagger \left(c_k + c_{k+1}\right) + H.c.\right] \\ &+ \sum_{k=1}^{n-1}\sum_{j} \eta_{k,j}\left(a_k^\dagger b_{k,j} + H.c.\right)\,. \end{aligned} \tag{27}$$

The first and second term in H^{free} are the free Hamiltonian of the cavity field and the two level atom inside each cavity, the third and forth term describe the free Hamiltonian of the fibers modes and of the environment of each fibers with mode frequencies ω_k^c, ω_k^a, ω_k^f and ω_k^e respectively. Also, the first term in the H^{int} describes the interaction between the cavity mode and the atom inside the cavity with the coupling strength f_k, the second term is the interaction between the cavity and the fibers modes with the coupling strength J_k and the last term is the interaction between the fibers and their bosonic baths with the coupling strength $\eta_{k,j}$. To write the above Hamiltonian we assumed the cavities are in the strong coupling regime, i.e., $f \gg \kappa^a, \kappa^c$, where κ^a and κ^c are the atomic and cavity decay rates respectively. So, we assume that both the decay rates are negligible compared with the coupling between the fibers and their environments.

The first two terms of H^{free} and the first term of H^{int} can be jointly diagonalized in the basis of polaritons [16]. On the resonance between atom and cavity, i.e., $\omega_k^c = \omega_k^a \equiv \omega_k$, the polaritonic states $|n\pm\rangle_k = (|g,n\rangle_k \pm |e,n-1\rangle_k)/\sqrt{2}$, with energies $E_k^\pm = n\omega_k \pm f_k\sqrt{n}$, are "created" by the operators $P_k^{(n\pm)\dagger} = |n\pm\rangle_k\langle g,0|$. Due to photon blockade and in the Mott phase, double or higher occupancy of the polaritonic states is prohibited, hence the only states to be considered are $|1,\pm\rangle_k$, with energies $\omega_k \pm f_k$ [17, 18]. Moreover, in rotating-wave approximation and interaction picture the inter-converting terms between different polaritons $P_k^{(1-)\dagger}P_{k+1}^{(1+)}$ and $P_k^{(1+)\dagger}P_{k+1}^{(1-)}$) in the interaction Hamiltonian are fast rotating and they average to zero. So, if initially polaritons are created solely by $P_k^{(1-)\dagger}$, which is possible by applying a global external laser to the atom-cavity system [16], the polaritonic state $|1,+\rangle$ will never appear. Then Hamiltonian (25), taking into account that each polariton can be treated as a two level system with ladder operator $\sigma_k^\dagger \equiv |1,-\rangle_k\langle g,0|$, can be rewritten as

$$\begin{aligned} H =& \sum_{k=1}^{n} (\omega_k - f_k)\sigma_k^\dagger\sigma_k + \sum_{k=1}^{n-1} \omega_k^f a_k^\dagger a_k + \sum_{k=1}^{n-1}\sum_{j} \omega_{k,j}^e b_{k,j}^\dagger b_{k,j} \\ &+ \sum_{k=1}^{n-1} J_k \left[a_k^\dagger\left(\sigma_k + \sigma_{k+1}\right) + H.c.\right] \\ &+ \sum_{k=1}^{n-1}\sum_{j} \eta_{k,j}\left(a_k^\dagger b_{k,j} + H.c.\right)\,. \end{aligned} \tag{28}$$

By adiabatic elimination of the fibers mode operators we obtain the effective Hamiltonian

$$\begin{aligned} H^{eff} =& \sum_{k=1}^{n} \omega_k'\sigma_k^\dagger\sigma_k + \sum_{k=1}^{n-1}\sum_{j} \omega_{k,j}'^e b_{k,j}^\dagger b_{k,j} \\ &+ \sum_{k=1}^{n-1} J_k'\left(\sigma_k^\dagger\sigma_{k+1} + \sigma_{k+1}^\dagger\sigma_k\right) \\ &+ \sum_{k=1}^{n-1}\sum_{j} \eta_{k,j}'\left[b_{k,j}^\dagger(\sigma_k + \sigma_{k+1}) + H.c.\right]\,, \end{aligned} \tag{29}$$

with

$$\omega'_k = \omega_k - f_k - \frac{2J_k^2}{\omega_k^f} + \frac{J_k^2}{\omega_k^f}\delta_{k,1} + \frac{J_k^2}{\omega_k^f}\delta_{k,n}\,, \tag{30}$$

$$J'_k = -\frac{J_k^2}{\omega_k^f}\,, \tag{31}$$

$$\omega'^e_k = \omega^e_k - \frac{2J_k^2}{\omega_k^f}\,, \tag{32}$$

$$\eta'_{k,j} = -\frac{J_k\eta_{k,j}}{\omega_k^f}\,. \tag{33}$$

This Hamiltonian describes a qubit chain with XY interaction where nearest-neighbor qubits dissipate into the same bosonic bath. By tracing out the bosonic baths, which are assumed at zero-temperature, and in the Born and Markov approximations, one finally obtains the master equation (4) describing the polariton system.

VI. CONCLUSION

We have presented a characterization of the steady-states of qubit networks where adjacent qubits are coupled both directly via a XY interaction, and indirectly via the coherent dissipation into the same bosonic bath at zero temperature. We have determined conditions allowing the distribution of steady-state entanglement. Rather interesting, the features of the steady-state entanglement are largely independent on the network topology and on the dynamical details (e.g., coupling constants and decay rates). The maximal amount of steady-state entanglement that can be achieved between two arbitrary qubits only depends on the size of the network and decreases linearly with it. The steady-state entanglement is also a function of the network initial state. Furthermore, our analytical results, supported by numerical evidences leads us to conjecture that the optimal initial state of the network is a symmetric, Dicke-like, state containing a single excitation.

An array of doped optical cavities coupled by optical fibers is also discussed as a physical implementation, at least for the case of a network reducing to a linear chain. Another system could be that of planar arrays of trapped electrons used for quantum information processing [19].

Finally, the performed study lends itself to consider extension from 2-body dissipation to n_d-body dissipation in n qubits network and to analyze the scaling properties of entanglement vs n_d/n. This is left for future explorations.

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